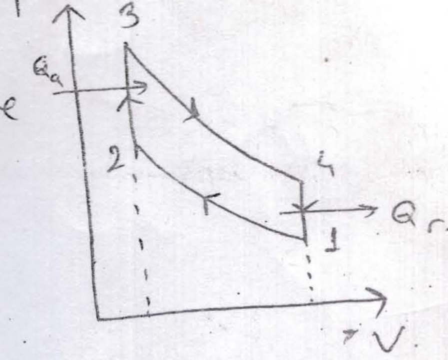
few extra questions.

S. Neupane

1 In an air standard Otto cycle, the ambient conditions determine the minimum temperature while the maximum temperature is determined by the design conditions and metallurgical considerations of the piston and cylinder. For fixed value of minimum temperature T_1 and maximum temperature T_3, show that for maximum work output, the compression ratio should have the value $r = \left(\frac{T3}{T1}\right)^{\frac{1}{2(\gamma-1)}}$



Sol^n:

workdone per kg of fluid in the cycle is given by

$$W = Q_a - Q_r = C_V(T_3 - T_2) - C_V(T_4 - T_1)$$

Again, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$T_2 = T_1 \cdot \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\therefore T_2 = T_1 \cdot (r_c)^{\gamma-1} \quad \left[r_c = \frac{V_1}{V_2} \right].$$

$|||^y$; $T_3 = T_4 (r_c)^{\gamma-1} \Rightarrow T_4 = T_3 / (r_c)^{\gamma-1}$

$$\therefore W = C_V \left[T_3 - T_2 - T_4 + T_1 \right].$$

$$= C_V \left[T_3 - T_1 \cdot (r_c)^{\gamma-1} - \frac{T_3}{(r_c)^{\gamma-1}} + T_1 \right]$$

when T_1 & T_3 are constant, $W = f(r_c)$

for maximum W; $\frac{dW}{dr_c} = 0$.

$$\frac{dW}{dr_c} = -T_1(\gamma-1) \cdot (r_c)^{\gamma-2} - T_3 \cdot r_c^{-\gamma} \cdot (1-\gamma) = 0.$$

$$T_1(\gamma-1) \cdot r_c^{\gamma-2} = T_3 r_c^{-\gamma} \cdot (\gamma-1)$$

$$T_3 r_c^{-\gamma} = T_1 \cdot r_c^{\gamma-2}$$

$$\frac{T_3}{T_1} = \frac{r_c^{\gamma-2}}{r_c^{-\gamma}} = r_c^{2\gamma-2} = r_c^{2(\gamma-1)}$$

$$\frac{T_3}{T_1} = r_c^{2(\gamma-1)}$$

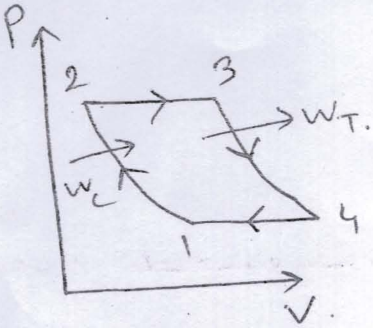
$$\therefore r_c = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2(\gamma-1)}} \quad \text{proved.}$$

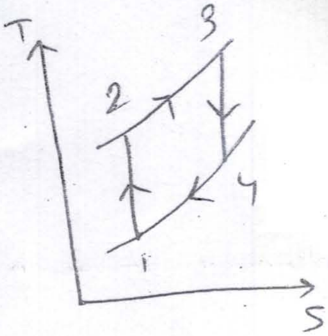
. Air enters the compressor of an ideal Brayton cycle at 100 *kPa*, 290 *K* with a volumetric flow rate of 4 m^3/s. The essure ratio for the cycle is 10 and the maximum temperature during the cycle is 1500 *K*. Determine:

i) the thermal efficiency of the cycle,

b) the fraction of work output that is consumed by the compressor, and

) the net power output.





$P_1 = 100\,kpa$

$T_1 = 290\,K$

$\dot{V} = 4\,m^3/s$

$r_p = \frac{P_2}{P_1} = 10$

$T_{max} = T_3 = 1500\,K$

$C_p = 1005\,J/kgK$

now, $PV = mRT$

$\therefore P\dot{V} = \dot{m}RT$

$100 \times 4 = \dot{m} \times 0.287 \times 290$

$\dot{m} = 4.8059\,kg/s$

(a) $(\eta_{th})_{Brayton} = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}$

$= 1 - \left(\frac{1}{10}\right)^{\frac{0.4}{1.4}}$

$= 0.48205$

$= 48.205\%$ Ans.

<u>for process 1-2;</u>

$T_1 P_1^{\frac{1-\gamma}{\gamma}} = T_2 P_2^{\frac{1-\gamma}{\gamma}}$

$\frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}}$

$\frac{290}{T_2} = (10)^{\frac{-0.4}{1.4}}$

$T_2 = 559.902\,K$

<u>for process 3-4;</u>

$T_3 P_3^{\frac{1-\gamma}{\gamma}} = T_4 P_4^{\frac{1-\gamma}{\gamma}}$

$\frac{T_4}{T_3} = \left(\frac{P_3}{P_4}\right)^{\frac{1-\gamma}{\gamma}}$

$\frac{T_4}{1500} = (10)^{\frac{-0.4}{1.4}}$

$T_4 = 776.921\,K$

$W_C = \dot{m}\,c_p(T_2 - T_1)$

$= 4.8059 \times 1.005 \times (559.902 - 290)$

$= 1303.607\,kW$

$W_T = \dot{m}\,c_p(T_3 - T_4)$

$= 4.8059 \times 1.005 \times (1500 - 776.921)$

$= 3492.4205\,kW$

(b) fraction of work consumed by compresser $= \frac{W_C}{W_T} = \frac{1303.607}{3492.42}$

$= 0.37326$

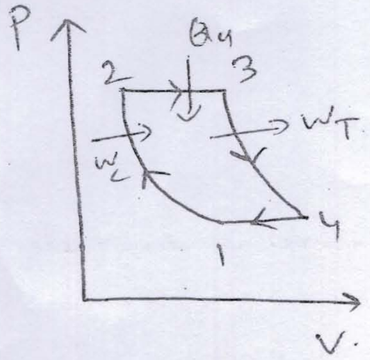
Ans.

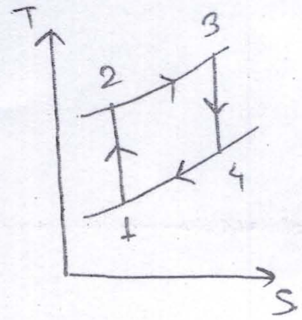
(c) net power output $= W_T - W_C$

$= 3492.42 - 1303.607$

$= 2188.813\,kW$ Ans.

. An ideal Brayton cycle has a pressure ratio of 12. The pressure and temperature at the compressor inlet are 100 kPa nd 27^0C respectively. The maximum temperature during the cycle is 1200^0C. If the mass flow rate of air is 8 kg/s, etermine the power output and efficiency of the cycle.





$r_p = \frac{P_2}{P_1} = 12$

$P_1 = 100\,kPa.$

$T_1 = 27^\circ C = 300K$

$T_{max} = T_3 = 1200^\circ C = 1473K$

$\dot{m} = 8\,kg/s.$

$W_{net} = ?$

$\eta_{Bry} = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}$

$= 1 - \left(\frac{1}{12}\right)^{\frac{0.4}{1.4}}$

$= 0.50834$

$= 50.834\%.$ Ans.

Again;

$T_1 P_1^{\frac{1-\gamma}{\gamma}} = T_2 P_2^{\frac{1-\gamma}{\gamma}}$

$\frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}}$

$T_2 = \frac{300}{12^{0.4/1.4}} = 610.181\,K.$

$Q_a = \dot{m} c_p (T_3 - T_2)$

$= 8 \times 1.005 (1473 - 610.18)$

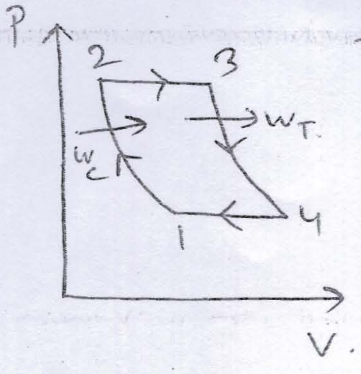
$= 6937.063\,kW.$

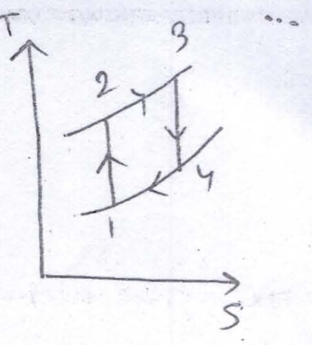
$\therefore W_{net} = \eta \times Q_a$

$= 0.50834 \times 6937.063$

$= 3526.38707\,kW.$ Ans.

5. The minimum and maximum temperatures during an ideal Brayton cycle are 300 K and 1200 K respectively. The pressure ratio is such that the net work developed is maximized. Determine:
a) the compressor and turbine work per unit mass of air, and
b) the thermal efficiency of the cycle.





$T_1 = 300\,K$

$T_3 = 1200\,K.$

$W_T = \dot m c_p(T_3 - T_4)$

$W_C = \dot m c_p(T_2 - T_1)$

$W_{net} = W_T - W_C = \dot m c_p[(T_3 - T_4) - (T_2 - T_1)].$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ $\qquad \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$

$T_2 = T_1 \times (r_p)^{\frac{\gamma-1}{\gamma}}$ $\qquad = (r_p)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = \frac{T_3}{(r_p)^{\frac{\gamma-1}{\gamma}}}.$

$W_{net} = \dot m c_p\left[\left(T_3 - \frac{T_3}{(r_p)^{\frac{\gamma-1}{\gamma}}}\right) - \left(T_1 (r_p)^{\frac{\gamma-1}{\gamma}} - T_1\right)\right]$

For W_{net} to be maximum;

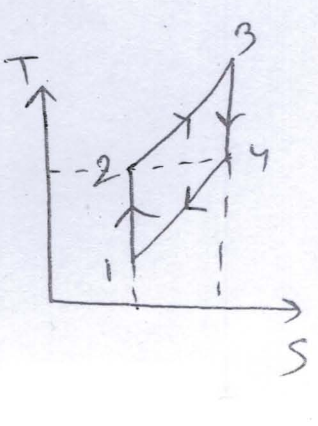
$\frac{dW_{net}}{dr_p} = 0.$

$-T_3\cdot(-)\frac{\gamma-1}{\gamma}\cdot(r_p)^{-\frac{(\gamma-1)}{\gamma}-1} - T_1\cdot\frac{\gamma-1}{\gamma}(r_p)^{\frac{\gamma-1}{\gamma}-1} = 0.$

$T_3\cdot\frac{\gamma-1}{\gamma}\cdot(r_p)^{\frac{1-2\gamma}{\gamma}} - T_1\cdot\frac{\gamma-1}{\gamma}(r_p)^{-1/\gamma} = 0.$

$\frac{T_3}{T_1} = \frac{(r_p)^{-1/\gamma}}{(r_p)^{\frac{1-2\gamma}{\gamma}}} = (r_p)^{\frac{2\gamma-2}{\gamma}}$

$\therefore r_p = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2\gamma-2}} = \left(\frac{1200}{300}\right)^{\frac{1.4}{0.8}} = 11.31$



$T_2 = 300 \times (11.31)^{\frac{0.4}{1.4}} = 600\,K.$

$T_4 = \frac{1200}{(11.31)^{\frac{0.4}{1.4}}} = 600\,K.$

$$\frac{W_T}{\dot{m}} = c_p(T_3 - T_4)$$

$$= 1.005(1200 - 600)$$

$$= 603\, KW. \quad \underline{Ans.}$$

$$\frac{W_c}{\dot{m}} = c_p(T_2 - T_1)$$

$$= 1.005(600 - 300)$$

$$= 301.5\, KW. \quad \underline{Ans}$$

$$\eta_{Bry} = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$= 1 - \left(\frac{1}{11.31}\right)^{\frac{0.4}{1.4}}$$

$$= 0.500046$$

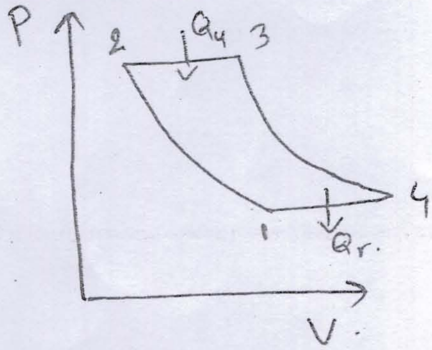
$$= 50\% \quad \underline{Ans}$$

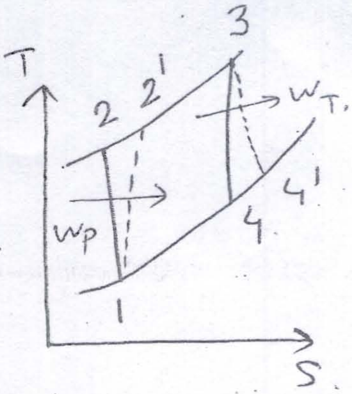
. The compressor and turbine of an ideal gas turbine each have isentropic efficiencies of 80 %. The pressure ratio is 10. The minimum and maximum temperatures are 300 K and 1500 K respectively. Determine:

a) the net work per kg of air,

b) the thermal efficiency of the cycle, and

c) Compare both of these for a cycle with ideal compressor and turbine.





$\eta_T = 0.8$

$\eta_p = 0.8$

$r_p = \frac{P_2}{P_1} = 10$

$T_1 = 300K$; $T_3 = 1500K$.

<u>for Turbine:</u>

$(W_T)_{actual} = h_3 - h_{4'}$

$(W_T)_{isen} = h_3 - h_4$.

$\eta_T = \frac{h_3 - h_{4'}}{h_3 - h_4}$

$0.8 = \frac{T_3 - T_{4'}}{T_3 - T_4}$.

Again;

$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$

$T_4 = 1500 \cdot \left(\frac{1}{10}\right)^{\frac{0.4}{1.4}}$

$= 776.921 K$.

$\therefore 0.8 = \frac{1500 - T_{4'}}{1500 - 776.921}$

$\therefore T_{4'} = 921.536 K$.

$\therefore (W_T)_{actual} = C_p (T_3 - T_{4'})$

$= 1.005 (1500 - 921.536)$

$= 581.355 KJ/kg$.

$(W_T)_{isen} = C_p (T_3 - T_4)$

$= 1.005 (1500 - 776.921)$

$= 726.69 KJ/kg$.

<u>for pump:</u>

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$T_2 = 300 \cdot (10)^{\frac{0.4}{1.4}}$

$= 579.209 K$.

$(W_P)_{actual} = h_{2'} - h_1$

$(W_P)_{isen} = h_2 - h_1$

for pump.

$$\eta_p = \frac{(W_P)_{isen}}{(W_P)_{actual}}$$

$$0.8 = \frac{h_2 - h_1}{h_{2'} - h_1}$$

$$0.8 = \frac{T_2 - T_1}{T_{2'} - T_1}$$

$$0.8 = \frac{579.201 - 300}{T_{2'} - 300}$$

$$T_{2'} = 649.011\,K.$$

$$(W_P)_{actual} = c_p(T_{2'} - T_1)$$
$$= 1.005(649.011 - 300)$$
$$= 350.75\,KJ/kg.$$

$$(W_P)_{isen} = c_p(T_2 - T_1)$$
$$= 1.005(579.209 - 300)$$
$$= 280.605.$$

$$(W_{net})_{actu.} = (W_T)_{actual} - (W_P)_{actual}$$
$$= 581.355 - 350.75$$
$$= 230.605\,KJ/kg.$$

$$(Q_a)_{actu} = c_p(T_3 - T_{2'})$$
$$= 1.005(1500 - 649.011)$$
$$= 855.24\,KJ/kg$$

$$(Q_a)_{isen} = c_p(T_3 - T_2)$$

$$(\eta_{Bry})_{actual} = \frac{W_{net}}{Q_a}$$
$$= \frac{230.605}{855.24}$$
$$= 26.964\%.$$

$$(W_{net})_{isen} = (W_T)_{isen} - (W_P)_{isen}$$
$$= 726.69 - 280.605$$
$$= 446.085\,KJ/kg$$

$$(\eta_{Bry})_{isen} = \frac{W_{net}}{Q_a}$$
$$= \frac{446.085}{925.402}$$
$$= 48.204\%$$

Also for ideal

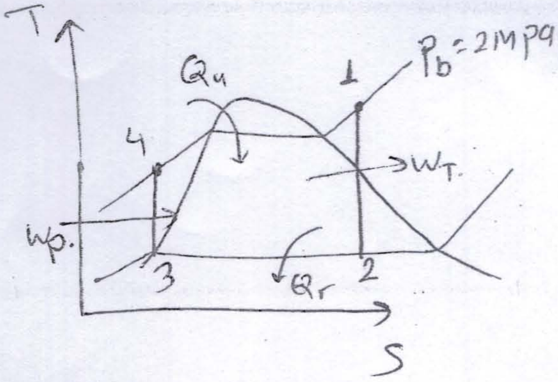
$$\eta_{Bry} = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}$$
$$= 1 - \left(\frac{1}{10}\right)^{\frac{0.4}{1.4}}$$
$$= 48.205\%$$

7. A Rankine cycle has a boiler working at a pressure of 2 MPa. The maximum and minimum temperatures during the cycle are 400^0C and 50^0C respectively. Determine the efficiency of the cycle and compare it with that of the Carnot cycle operating between the same temperature limits.

$P_b = 2\,MPa,\ T_1 = 400^\circ C.$

$P_c = P_{sat}$ at $T = 50^\circ C.$

$= 12.344\ kPa.$



$h_1 = 3247.5\ kJ/kg.$

$s_1 = 7.1269\ kJ/kgK.$

Process 1-2:

$s_1 = s_2$

$7.1269 = (s_f + x_2 s_{fg})_{T=50^\circ c}$

$7.1269 = 0.7037 + x_2 (7.3708)$

$x_2 = 0.8714$

$\therefore h_2 = (h_f + x_2 h_{fg})_{T=50^\circ c}$

$= 209.33 + 0.8714 \times 2381.9$

$= 2285.0097\ kJ/kg.$

$h_3 = (h_f)_{T=50^\circ c}$

$= 209.33\ kJ/kg.$

$v_3 = (v_f)_{T=50^\circ c}.$

$= 0.0010120\ m^3/kg.$

$\therefore h_4 = h_3 + v_3 (P_b - P_c)$

$= 209.33 + 0.001012 (2000 - 12.344)$

$= 211.341\ kJ/kg.$

$\eta_{rankine} = 1 - \frac{Q_r}{Q_a}$

$= 1 - \frac{h_2 - h_3}{h_1 - h_4}$

$= 1 - \frac{2075.67}{3036.159}$

$= 31.63\%.$ As.

$\eta_{carnot} = 1 - \frac{T_2}{T_1}$

$= 1 - \frac{323}{673}$

$= 52\%.$

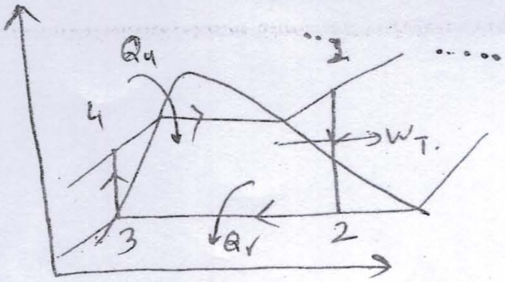
As

. An ideal Rankine cycle operates between a boiler pressure of 4 MPa and a condenser pressure of 10 kPa. The exit eam from the turbine should have a quality of 96 % and the power output of the turbine should be 80 MW. Determine

ı) the minimum boiler exit temperature,

) the efficiency of the cycle, and

:) the mass flow rate of steam.



$P_b = 4000 kPa$.

$P_c = 10 kPa$.

$x_2 = 0.96$

$W_T = 80 MW$.

$h_2 = (h_f + x_2 h_{fg})_{P=10kPa}$

$= 191.83 + 0.96 \times 2392$

$= 2488.15 \, kJ/kg$.

$s_2 = (s_f + x_2 s_{fg})_{P=10kPa}$

$= 0.6493 + 0.96 \times 7.4989$

$= 7.84824 \, kJ/kgK$

$\therefore s_1 = s_2 = 7.84824 \, kJ/kgK$.

$\therefore$ Boiler outlet temp $= 800°C$ (from superheated table).

$\therefore h_1 = 4141.7 \, kJ/kg$

$h_3 = (h_f)_{P=10kPa}$

$= 191.83 \, kJ/kg$.

$v_3 = (v_f)_{P=10kPa}$

$= 0.001010 \, m^3/kg$.

$h_4 = h_3 + v_3 (P_b - P_c)$

$= 191.83 + 0.001010(3990)$

$= 195.859 \, kJ/kg$

$W_T = \dot{m}(h_1 - h_2)$

$80 \times 10^3 = \dot{m}(4141.7 - 2488.15)$

$\dot{m} = 48.3807 \, kg/s$.

$\eta_{rankine} = 1 - \dfrac{Q_r}{Q_a}$

$= 1 - \dfrac{h_2 - h_3}{h_1 - h_4}$

$= 1 - \dfrac{2296.32}{3945.841}$

$= 41.804 \%$.

As

). Saturated vapor enters into a turbine of an ideal Rankine cycle at 10 MPa and saturated liquid exits the condenser at) kPa. The power output of the cycle is 120 MW. Determine:

) the thermal efficiency of the cycle,

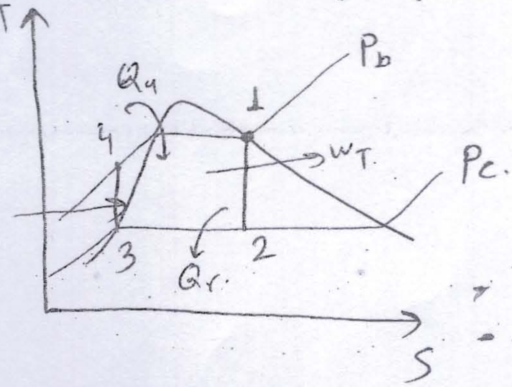
) the back work ratio,

) the mass flow rate of steam,

) the rate at which heat is supplied to the boiler,

) the rate at which heat is rejected from the condenser, and

) the mass flow rate of condenser cooling water, if the cooling water enters at 20^0C and exits at 35^0C. [Take specific heat water as 4.18 kJ/kgK].



$P_b = 10000\ kPa$

$P_c = 10\ kPa$

$W_{net} = 120\ MW$

$h_1 = (h_g)_{p=P_b}$

$= 2724.5\ kJ/kg$

$s_1 = (s_g)_{p=P_b}$

$= 5.6139\ kJ/kgK$

$s_1 = s_2$

$5.6139 = (s_f + x_2 s_{fg})_{p=P_c}$

$5.6139 = 0.6493 + x_2 \times 7.4989$

$x_2 = 0.66204$

$h_2 = h_f + x_2 h_{fg}$

$= 191.83 + 0.66204 \times 2392$

$= 1775.438\ kJ/kg$

$h_3 = (h_f)_{p=10}$

$= 191.83\ kJ/kg$

$v_3 = (v_f)_{p=10kPa}$

$= 0.001010\ m^3/kg$

$h_4 = h_3 + v_3(P_b - P_c)$

$= 191.83 + 0.001010 \times 9990$

$= 201.9199\ kJ/kg$

$\eta = 1 - \frac{Q_r}{Q_a}$

$= 1 - \frac{h_2 - h_3}{h_1 - h_4}$

$= 1 - \frac{1583.608}{2522.58}$

$= 37.22\%$

$W_T = \dot{m}[h_1 - h_2]$

$W_T = \dot{m} \times 949.062$

$W_P = \dot{m}[h_4 - h_3]$

$= \dot{m} \times 10.0899$

$W_{net} = \dot{m}[949.062 - 10.0899]$

$\dot{m} = 127.79\ kg/s$

$\therefore W_T = 121.289\ MW$

$W_P = 1.28938\ MW$

Backwork $= \frac{W_P}{W_T}$

$= 1.06 \times 10^{-2}$

$$Q_a = \dot{m}[h_1 - h_4)$$

$$= 127.79(2724.5 - 201.9199)$$

$$= 322.360 \text{ MW.}$$

$$Q_r = \dot{m}[h_9 - h_3]$$

$$= 127.79(1775.438 - 191.83)$$

$$= 202.369 \text{ MW.}$$

$$Q_r = \dot{m}_{wat}(35-20) \times c_p.$$

$$202369 = \dot{m}_{wat} \times 15 \times 4.18$$

$$\dot{m}_{water} = 3227.575 \text{ kg/s.}$$

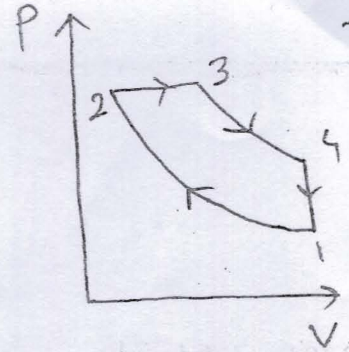
5. An air standard diesel cycle has a compression ratio of 22 and expansion ratio of 11. Determine its cut off ratio and the efficiency.

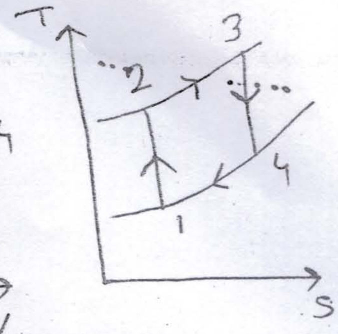
$r_c = \frac{V_1}{V_2} = 22.$

$r_e = 11 = \frac{V_4}{V_3}.$

$\beta = \frac{V_3}{V_2} = ?$

$\eta = ?$





soln.

$\cancel{\frac{r_e}{r_c} = \frac{V_3/V_4}{V_1/V_2}}$

$\cancel{\frac{11}{22} = \frac{V_3}{V_2} \times \frac{V_2}{V_1}}$

$$\frac{r_c}{r_e} = \frac{\frac{V_1}{V_2}}{\frac{V_4}{V_3}} = \frac{V_1}{V_4} \times \frac{V_3}{V_2}$$

$$= 1 \times \beta$$

$$\therefore \frac{r_c}{r_e} = \beta$$

$$\Rightarrow \frac{22}{11} = \beta$$

$\therefore \beta =$ cutoff ratio $= 2.$

$$\eta_{diesel} = 1 - \left(\frac{1}{r_c}\right)^{\gamma-1}\left[\frac{1}{\gamma} \cdot \frac{\beta^{\gamma} - 1}{\beta - 1}\right]$$

$$= 1 - \left(\frac{1}{22}\right)^{0.4}\left[\frac{1}{1.4} \frac{2^{1.4} - 1}{2 - 1}\right]$$

$$= 65.999\ \%.$$

few extra questions   S Neupane.

3. A heat pump maintains a room at a temperature of 20^0C when the surrounding is at 5^0C. The rate of heat loss from the room is estimated to be 0.6 kW per degree temperature difference between inside and outside. If the electricity costs $Rs\ 10/kWh$, determine the minimum theoretical cost per day.

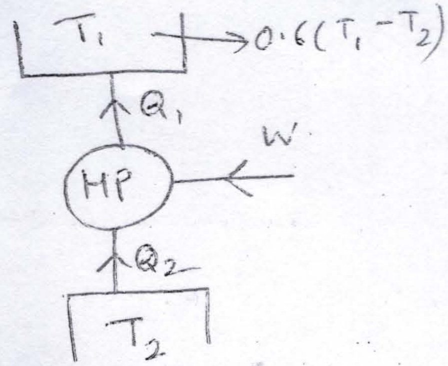
soln:

$T_1 = 20^\circ C = 293K$

$T_2 = 5^\circ C = 278K$

$Q_1 = 0.6(T_1 - T_2)$

Cost = Rs 10/kwh.



$$\left\{(COP)_{HP}\right\}_{Carnot} = \frac{T_1}{T_1 - T_2}$$

$$= \frac{293}{293-278}$$

$$= \frac{293}{15}$$

$$= 19.53333$$

for minimum cost;

$$\frac{T_1}{T_1 - T_2} = \frac{Q_1}{W}$$

$$\frac{293}{15} = \frac{0.6 \times 15}{W}$$

$$\therefore W = 0.46075 kW$$

$\therefore$ minimum cost per day $= 0.46075 \times 10 \times 24$

$= 110.5802$

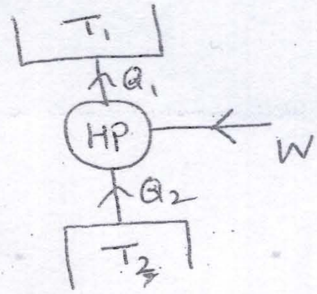
$=$ Rs 110.5802 Ans.

4. A heat pump heats a house in the winter and then reverses to cool it in the summer. The room temperature should be $22^{o}C$ in the winter and $26^{o}C$ in the summer. Heat transfer through the walls and ceilings is estimated to be $3000\ kJ/h$ per degree temperature difference between the inside and outside.

(a) Determine the power required to run it in the winter when the outside temperature decrease to $0^{o}C$.

(b) If the unit is run by the same power as calculated in (a), throughout the year, determine the maximum outside summer temperature for which the house can be maintained at $26^{o}C$.

(a) for winter



$T_1 = 22^{\circ}C = 295\,K$

$T_2 = 0^{\circ}C = 273\,K$

$Q_1 = 3000(T_1 - T_2)$

$W = ?$

Now: $\dfrac{T_1}{T_1 - T_2} = \dfrac{Q_1}{W}$

$W = \dfrac{Q_1(T_1 - T_2)}{T_1}$

$= \dfrac{3000 \times 22 \times 22}{295}$

$= \dfrac{4922.03}{3600}$

$W = 1.3672\ kW$.

for summer:

(b) $T_2 = 26^{\circ}C = 299\,K$

$T_1 = ?$

Now; $\dfrac{T_2}{T_1 - T_2} = \dfrac{Q_2}{W}$

$\dfrac{T_2}{T_1 - T_2} = \dfrac{3000(T_1 - T_2)}{4922.03}$

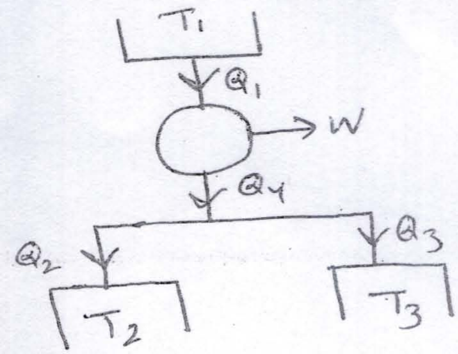
$(T_1 - T_2)^2 = \dfrac{299 \times 4922.03}{3000}$

$T_1 - T_2 = 22.148$

$T_1 = 321.148\,K$

$\therefore T_1 = 48.148^{\circ}C$.

7. A reversible engine working in a cycle takes 4800 kJ/min of heat from a source at 800 K and develops 20 kW power. The engine rejects heat to two reservoirs at 300 K and 360 K. Determine the heat rejected to each sink.



$Q_1 = 4800\,kJ/min = 80\,kW$

$T_1 = 800K$

$W = 20KW$

$T_2 = 300K$

$T_3 = 360K$

$\eta = \frac{W}{Q_1}$

$= \frac{20}{80}$

$= 1/4$

$\eta = 1 - \frac{Q_2 + Q_3}{Q_1}$

$\frac{1}{4} = 1 - \frac{Q_2 + Q_3}{Q_1}$

$Q_2 + Q_3 = 60\,kW.$

Again, using clausius inequality;

$\oint \frac{\delta Q}{T} = 0.$

$\frac{Q_1}{T_1} + \frac{(-)Q_2}{T_2} + \frac{-Q_3}{T_3} = 0.$

$\frac{80}{800} - \frac{60 - Q_3}{300} - \frac{Q_3}{360} = 0.$

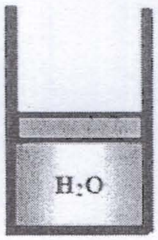
$\frac{1}{10} - \frac{1}{5} + \frac{Q_3}{300} - \frac{Q_3}{360} = 0.$

$Q_3\left(\frac{1}{300} - \frac{1}{360}\right) = \frac{1}{10}.$

$Q_3 = 180\,kW.$ Ans.

$\therefore Q_2 = -120\,kW.$

9. A piston cylinder device shown in **Figure A2.9** contains 1 kg of water at saturated vapor state 500 kPa. It is cooled so that its volume reduces to half of the initial volume because of heat transfer to the surrounding at 20°C. Determine the total entropy generated during the process.



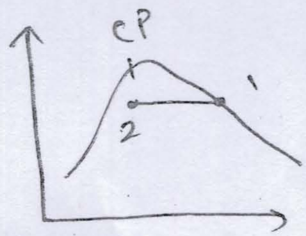


Figure A2.9

$m_w = 1\,Kg$

$v_1 = v_g = 0.3749\, m^3/kg$

$P_1 = 500\,kPa$

$v_2 = \frac{v_1}{2} = 0.18745$

$P_2 = P_1 = 500\,kPa$

$v_2 = (v_f + x_2 v_{fg})_{p=500kpa}$

$0.18745 = 0.001093 + x_2 \times 0.3738$

$x_2 = 0.498547$

$s_1 = (s_g)_{p=500kpa} = 6.8214$

$s_2 = (s_f + x_2 s_{fg})_{p=500}$

$= 1.8610 + 0.498547 \times 4.9604$

$= 4.3339\ KJ/KgK$

$u_1 = (u_g)_{p=500kpa} = 2561.2\,KJ/kg.$

$u_2 = (u_f + x_2 u_{fg})_{p=500kpa} = 1597.748\,KJ/kg.$

$h_1 = (h_g)_{p=500kpa} = 2748.6\,KJ/kg.$

$h_2 = (h_f + x_2 h_{fg})_{p=500kpa} = 640.38 + 0.498547 \times 2108.2$

$= 1691.4167\,KJ/kg.$

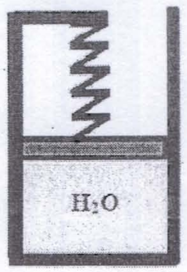
$h_1 - h_2 = -1057.18\,KJ.$

$S_{gen} = (s_2 - s_1) - \Sigma Q/T$

$= (4.3339 - 6.8214) - \left(\frac{-1057.18}{293}\right)$

$= 1.1206\ KJ/K.$ An

10. A piston cylinder device loaded with a linear spring as shown in **Figure A2.10** contains 0.5 kg of water at 100 kPa and 25°C. Heat is transferred from a source at 750°C until water reaches to a final state of at 1000 kPa and 600oC. Determine the total entropy generated during the process.



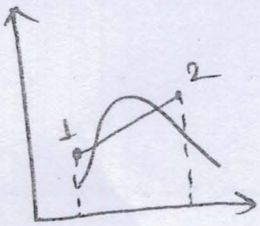


Figure A2.10

$m_w = 0.5\,kg$

$P_1 = 100\,kPa$

$T_1 = 25°C$

$P_2 = 1000\,kPa$

$T_2 = 600°C$

At $P_1 = 100\,kPa,\ T_1 = 25°C$

$T_{sat} = 99.632$

$\therefore v_1 = 0.001043\,m^3/kg$

At $P_2 = 1000\,kPa,\ T_2 = 600°C$

$v_2 = 0.4011\,m^3/kg$

$s_1 = 1.3027\,kJ/kgK$

$s_2 = 8.0292\,kJ/kgK$

$(\delta w)_{1-2} = \frac{1}{2} \times (v_2 - v_1) \times m \times (P_1 + P_2)$

$= \frac{1}{2} \times 0.400057 \times 0.5 \times 1100$

$= \cancel{44.01425\,kJ}$

$= 110.01\,kJ$

$u_1 = 417.45\,kJ/kg$

$u_2 = 3297\,kJ/kg$

$(u_2 - u_1) \times m = 1439.795\,kJ$

$\therefore (\delta q)_{1-2} = 1439.795 + \cancel{44.01425}$ 110.01

$= \cancel{1483.80\,kJ}$

$= 1549.805\,kJ$

$S_{gen} = (s_2 - s_1) \times m - \frac{\delta q}{T}$

$= (8.0292 - 1.3027) \times 0.5 - \frac{\cancel{1483.8}\ 1549.8}{1023}$

$= 3.36325 - \cancel{1.4504}\ 1.5149$

$= \cancel{1.9128\,kJ}$

$= 1.848\,kJ/K$

11. A piston cylinder device shown in **Figure A2.11** contains 1.5 kg of water initially at 100 kPawith 10 % of quality. The mass of the piston is such that a pressure of 500 kPais required to lift the piston. Heat is added to the system from a source at 500°C until its temperature reaches400°C. Determine the total entropy generation during the process.

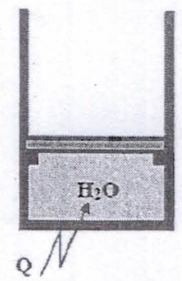
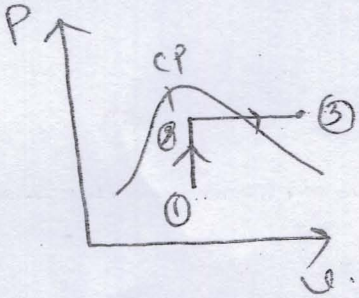


Figure A2.11



$m_w = 1.5\,kg$

$P_1 = 100\,kPa$

$x_1 = 0.1$

$P_2 = 500\,kPa$

$T_{final} = 400°c$

$v_1 = (v_f + x_1 v_{fg})_{P_1 = 100kPa}$

$= 0.001043 + 0.1 \times 1.6933$

$= 0.170373\,m^3/kg$

$\therefore v_2 = v_1 = 0.170373;\ P_2 = 500kPa$

$(v_f)_{500} = 0.001093\ m^3/kg$

$(v_g)_{500} = 0.3749\ m^3/kg.$

$\therefore$ state 2 is at 2 phase.

$\therefore v_2 = v_f + x_2 v_{fg}$

$0.170373 = 0.001093 + x_2 \times 0.3738$

$\therefore x_2 = 0.4528$

for state 3, $P_3 = 500kPa, T_3 = 400°c$

$\therefore v_3 = 0.7407\,m^3/kg.$

$s_3 = 8.0968\,kJ/kgK.$

Again,

$u_1 = (u_f + x_1 u_{fg})_{P=100kPa}$

$= 418.96 + 0.1 \times 2087.1$

$= 627.67\,kJ/kg$

$u_2 = (u_f + x_2 u_{fg})_{P=500kPa}$

$= 639.84 + 0.4528 \times 1921.4$

$= 1509.849\,kJ/kg.$

$h_2 = (h_f + x_2 h_{fg})_{P=500kPa}$

$= 640.38 + 0.4528 \times 2108.2$

$= 1594.97296\,kJ/kg.$

$s_1 = (s_f + x_1 s_{fg})_{P=100kPa}$

$= 1.3027 + 0.1 \times 6.0562$

$= 1.90832\,kJ/kgK.$

QN 1 Contd.

$$\therefore (\delta Q)_{1-3} = (\delta Q)_{1-2} + (\delta Q)_{2-3} \ldots\ldots$$

$$= \left[(u_2 - u_1) + (h_3 - h_2)\right] m_w$$

$$= \{(1509.849 - 627.67) + (3483.95 - 1594.972)\} \times 1.5$$

$$= 4156.195 \text{ KJ.}$$

$$\therefore (s_3 - s_1) m = \frac{\delta Q}{T} + S_{gen}$$

$$(8.0968 - 1.90832) \times 1.5 = \frac{4156.195}{773} + S_{gen}$$

$$9.28272 = 5.3767 + S_{gen}$$

$$\therefore S_{gen} = 3.90601 \text{ KJ/K.}$$

VVI

12. Water is contained in a piston cylinder device with two set of stops as shown in **Figure A2.12** is initially at 1 MPa and 400°C. The limiting volumes are V_{min}=1 $m3$ and V_{max}=2 $m3$. The weight of the piston is such that a pressure of 400 kPa is required to support the piston. The system is cooled to 100°C by allowing system to reject heat to the surroundings at 25°C. Sketch the process on $P-v$ and $T-v$ diagrams and determine the total entropy generated during the process.

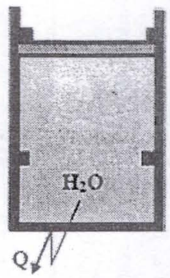
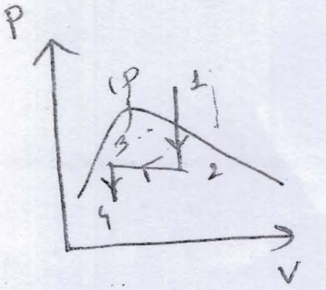
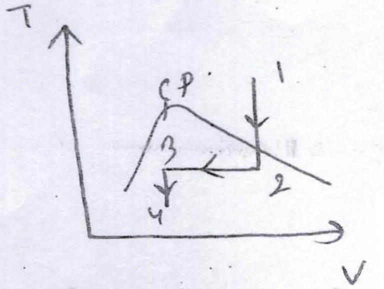


Figure A2.12





$P_1 = 1$ MPa

$T_1 = 400°C$

$V_1 = 2\ m^3$

$V_{final} = 1\ m^3$

$\cancel{V_3} =$

$T_{final} = 100°C$.

At $P_1 = 1$ MPa, $T_1 = 400°C$

$v_1 = 0.3066$ $\quad \therefore m = \frac{V_1}{v_1}$

$u_1 = 2957.2$ $\quad = \underline{6.523 kg}$.

At $P_2 = 400$ kPa, $v_2 = 0.3066$.

2 phase

$v_2 = v_f + x_2 v_{fg}$.

$0.3066 = 0.001084 + x_2 \times 0.4614$

$x_2 = 0.66214$.

$u_2 = u_f + x_2 u_{fg})_{P=400kPa}$

$= 604.47 + 0.66214 \times 2553.5$

$= 2295.24$ kJ/kg

$h_2 = h_f + x_2 h_{fg})_{P=400kPa}$

$= 604.91 + 0.66214 \times 2133.6$

$= 2017.65$ kJ/kg.

At $T = 100°C$, $v_f = \frac{1}{6.523}$

$= 0.1533\ m^3/kg$.

At 400 kPa, $v_3 = 0.1533\ m^3/kg$

2 phase.

$v_3 = v_f + x_3 v_{fg})_{P=400kPa}$.

$0.1533 = 0.001084 + x_3 \times 0.4614$

$x_3 = 0.3299$.

Again;

at $T = 100°C$, $v_4 = 0.1533$

2 phase

$v_4 = v_f + x_4 v_{fg})_{T=100°C}$

$0.1533 = 0.001043 + x_4 \times 1.6726$

$x_4 = 0.09103$

$\therefore u_4 = (u_f + x_4 u_{fg})_{T=100}$

$= 418.96 + 0.09103 \times 2087.1$

$= 608.94899$ kJ/kg

$\therefore (\delta Q)_{1-4} = (du)_{1-4} + \delta W$

$= (u_4 - u_1) + P_2(V_3 - V_2)$

$= [608.94899 - 2957.2] \times m$

$+ 400(1-2)$

$\delta Q = -15717.64$ kJ.

2 Contd

$$s_4 = (s_f + x_4 s_{fg})_{T=100^\circ}$$

$$= 1.3069 + 0.09103 \times 6.0476$$

$$= 1.857413 \text{ KJ/kgK.}$$

$$s_1 = 7.4648 \text{ KJ/kgK.}$$

Now:

$$(s_4 - s_1) m = \frac{Q}{T} + S_{gen}$$

$$(1.857413 - 7.4648) \times 6.523 = -\frac{15717.6}{298} + S_{gen}$$

$$\therefore S_{gen} = 16.166 \text{ KJ/K} \quad \text{Ans}$$

13. Steam enters a nozzle at 1.5 MPaand 300oC and with a velocity of 50 m/s, undergoes a reversible adiabatic process and exits at 200 kPa. Determine the exit velocity.

$P_1 = 1.5 MPa$

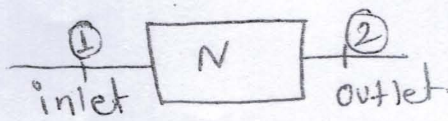
$T_1 = 300°c.$

$V_1 = 50 m/s.$

reversible adiabatic.

$P_2 = 200 kPa.$

$V_2 = ?$



from: $P_1 = 1500 kPa, T_1 = 300°c.$

$T_{sat} = 198.33$

∴ from table

$h_1 = 3036.9 \; KJ/Kg.$

$s_1 = 6.9168 \; KJ/KgK$

Again, $P_2 = 200 kPa, S_2 = 6.9168 \; KJ/Kg K$

2 phase.

$$S_2 = (S_f + x_2 S_{fg})_{P = 200kPa}$$

$$6.9168 = 1.5304 + x_2 \times 5.5968$$

$$x_2 = 0.962407$$

Again,

$$h_2 = (h_f + x_2 h_{fg})_{P = 200kPa.}$$

$$= 504.8 + 0.962407 \times 2201.7$$

$$= 2623.73 \; KJ/Kg.$$

Now;

$$\dot{m}\left[(h_1 - h_2) + \tfrac{1}{2}(v_1^2 - v_2^2)\right] = 0.$$

$$(3036.9 - 2623.73) + \frac{1}{2000}(50^2 - v_2^2) = 0$$

$$413.168 + 1.25 - \frac{v_2^2}{2000} = 0.$$

$$v_2 = 910.404 \; m/s.$$

14. A compressor receives air at 100 kPaand 27°C and requires a power input of 60 kW. If the mass flow rate of the air is 0.1 kg/s, determine the maximum exit pressure of the compressor. [Take $\gamma=1.4$, $c_P=1005$ J/kgK]

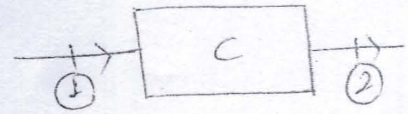
$P_1 = 100$ kpa

$T_1 = 27°c = 300K$

$\dot{W}_{cv} = -60kW$

$\dot{m} = 0.1 kg/s$

$P_2 = ?$



Now,

$$W_{cv} = \dot{m}[h_1 - h_2]$$

$$-60\times10^3 = 0.1 \times 1.005 (T_1 - T_2)$$

$$T_1 - T_2 = -597.014 K$$

$$300 - T_2 = -597.014K$$

$$T_2 = 897.014K$$

Again, for isentropic condition

$$Pv^\gamma = K$$

$$\text{or, } P\cdot\left(\frac{RT}{P}\right)^\gamma = K \quad (Pv = RT)$$

$$\text{or, } P^{-\gamma+1}\cdot T^\gamma = K$$

$$\text{or, } P\cdot T^{\gamma/1-\gamma} = K$$

$$P_1 T_1^{\frac{\gamma}{1-\gamma}} = P_2 T_2^{\frac{\gamma}{1-\gamma}}$$

$$\text{or, } P_2 = P_1\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{1-\gamma}}$$

$$= 100\times\left(\frac{300}{897.01}\right)^{\frac{1.4}{-0.4}}$$

$P_2 = 4622.384$ kpa is maximum pressure.

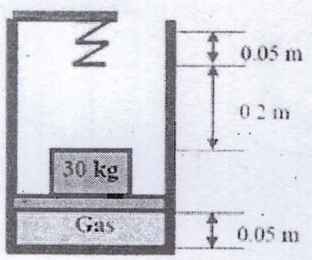
few extra questions. S. Neupane.

4. A gas enclosed by a piston shown in **Figure A1.4** starts to expand due to heating. The initial movement of 0.2 m is restrained by a fixed mass of 30 kg and the final 0.05 m is restrained both by the mass and a spring of stiffness 10 kN/m. The cross sectional area of the piston is 0.15 m2 and the atmospheric pressure is 100 kPa.

(a) Neglecting the mass of the spring and the piston sketch a P-V diagram of the process.

(b) Calculate the work during the initial 0.2 m movement.

(c) Calculate the total work done.



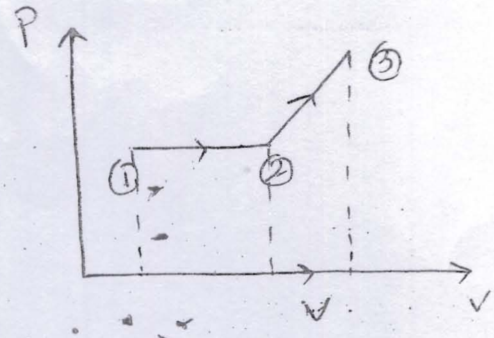


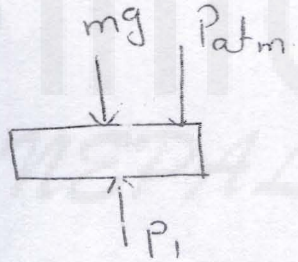
Figure A1.4

$m = 30\,kg$

$V_1 = 0.15 \times 0.05 = 7.5 \times 10^{-3} m^3$

$V_2 = 0.25 \times 0.15 = 37.5 \times 10^{-3} m^3$

$V_3 = 0.30 \times 0.15 = 45 \times 10^{-3} m^3$

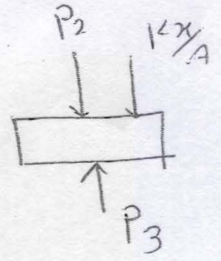


$\therefore P_1 = \dfrac{mg}{A \times 10^3} + P_{atm}$

$= \dfrac{30 \times 9.81}{0.15 \times 10^3} + 100$

$= 101.962\,kPa$

$P_2 = P_1 = 101.962\,kPa$



$P_3 = P_2 + \dfrac{kx}{A}$

$= 101.962 + \dfrac{10 \times 0.05}{0.15}$

$= 105.295\,kPa$

Work done during initial 0.2m movement

is $W_{1-2} = P_1(V_2 - V_1)$

$= 101.962(37.5 \times 10^{-3} - 7.5 \times 10^{-3})$

4 - contd

Total workdone $= W_{1-2} + W_{2-3}$

$= 3.1588 + \frac{1}{2} \times (P_2 + P_3) \cdot (V_3 - V_2)$

$= 3.1588 + \frac{1}{2} \times (101.962 + 105.295) \times 7.5 \times 10^{-3}$

$= 3.1588 + 0.7772$

$= 3.936\ kJ$ Ans

5. The frictionless piston shown in **Figure A1.5** has a mass of 20 kg and a cross sectional area of 78.48 cm2. Heat is added until the temperature reaches 400oC. If the quality of the H2O at the initial state is 0.2, determine:

(a) the initial pressure,

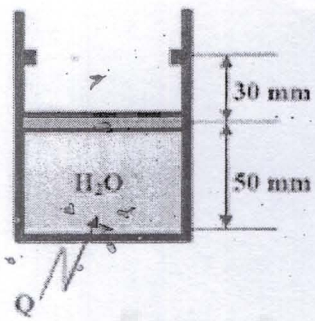
(b) the mass of H2O,

(c) the quality of the system when the piston hits the stops,

(d) the final pressure, and

(e) the total work transfer.

[Take Patm= 100 kPa, g = 9.81 m/s2]



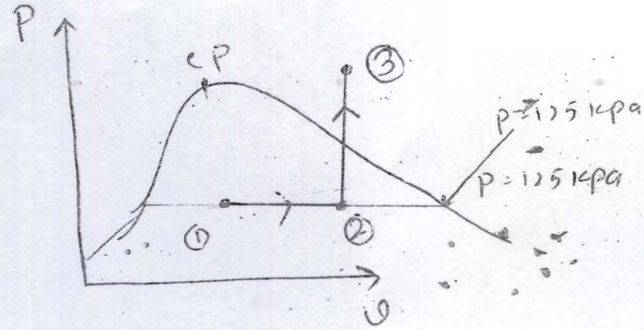


Figure A1.5

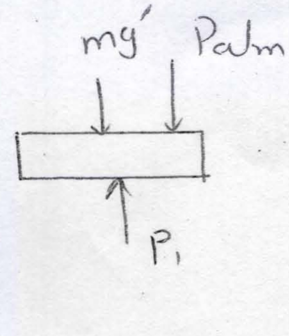
$m_p = 20\,kg$ $\quad P_1 = ?$

$A = 78.48\,cm^2$ $\quad mass_{H_2O} = ?$

$T_{final} = 400^\circ C$ $\quad$ quality when piston hit stop $x_2 = ?$

$x_1 = 0.2$ $\quad P_{final} = ?$

$W_{total} = ?$



$$P_1 = \frac{mg}{A} + P_{atm}$$

$$= \frac{20 \times 9.81}{78.48 \times 10^{-4} \times 10^3} + 100$$

$$P_1 = 125\,kPa$$ Ans

Q5 Contd

$V_1 = A \times h_1$
$= 78.48 \times 10^{-4} \times 50 \times 10^{-3}$
$= 3.924 \times 10^{-4}\, m^3$

$V_2 = A \times h_2$
$= 78.48 \times 10^{-4} \times 80 \times 10^{-3}$
$= 6.2784 \times 10^{-4}\, m^3$

Since $x_1 = 0.2$;

$v_1 = (v_f + x_1 v_{fg})_{P=125kPa}$
$= 0.001048 + 0.2 \times 1.3748$
$= 0.276\, m^3/kg$

$\therefore$ mass of $H_2O = \frac{V_1}{v_1} = \frac{3.924 \times 10^{-4}}{0.276} = 1.42 \times 10^{-3} kg$
$= 0.00142\, kg$ Ans

Now; when piston hit stop

$v_2 = \frac{V_2}{m} = \frac{6.2784 \times 10^{-4}}{0.00142}$
$= 0.4416\, m^3/kg$

$v_2 = (v_f + x_2 v_{fg})_{P=125kPa}$

$0.4416 = 0.001048 + x_2 \times 1.3748$

$\therefore x_2 = 0.3204$ Ans

Temperature at this instant $T_2 = 105.99°C$
It suggest that heating is continued till temperature reaches 400°c at constant volume.

$\therefore T = 400°C$, $v_3 = v_2 = 0.4416\, m^3/kg$.

Q5
contd.

Therefore from superheated table

$T = 400°C$ $\quad v = 0.5137\ m^3/kg$ at $p = 600\ kPa$

$v = 0.3843\ m^3/kg$ at $p = 800\ kPa$

$v = 0.4416\ m^3/kg$ $p = ?$

Now using linear interpolation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$P = 600 + \frac{800 - 600}{0.3843 - 0.5137}(0.4416 - 0.5137)$$

$P_{final} = 711.437\ kPa$ Ans.

$$W_{total} = W_{1-2} + W_{2-3}$$

$$= P_1(v_2 - v_1) \times m + 0$$

$$= 125(0.4416 - 0.276) \times 0.00142$$

$$= 0.029394\ kJ$$ Ans.

7. A piston cylinder arrangement shown in **Figure A1.7** contains water initially at a pressure of 1 MPa and a temperature of 400°C. Heat is transferred from the system to the surroundings until its pressure drops to 100 kPa. Sketch the process on P-v and T-v diagrams and determine:

(a) the mass of H_2O in the system, and

(b) the total work transfer.

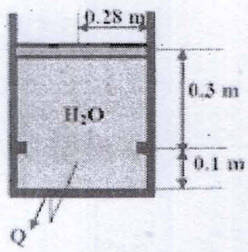
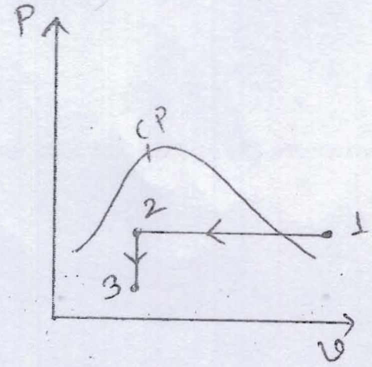
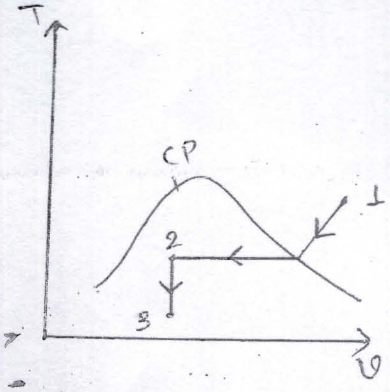


Figure A1.7





$P_1 = 1\,\text{mpa} = 1000\,\text{kpa}$.

$T_1 = 400°c$.

$P_{final} = 100\,\text{kpa}$.

$P = 1000\,\text{kpa}$ and $T = 400°c$

$T_{sat} = 179.92°c$.

$T > T_{sat}$, superheated.

$v_1 = 0.3066\,m^3/kg$.

From the figure:

$V_1 = \pi r^2 \times h_1$

$= \pi \times (0.28)^2 \times (0.3+0.1)$

$= 0.0985\,m^3$.

$V_2 = \pi r^2 \times h_2$

$= \pi \times (0.28)^2 \times 0.1$

$= 0.02463\,m^3$.

$\therefore$ mass of water $(m) = \dfrac{V_1}{v_1} = 0.3213\,kg$

P_2 = pressure at the instant when piston just touch the stop

$\therefore P_2 = 1000\,\text{kpa}$.

$v_2 = \dfrac{V_2}{m} = \dfrac{0.02463}{0.3213} = 0.07664\,m^3/kg$.

At $P_2 = 1000\,\text{kpa}$, $v_2 = 0.07664\,m^3/kg$

So, 2 phase.

$\therefore v_2 = (v_f + x_2 v_{fg})_{P=1000\,\text{kpa}}$

$0.07664 = 0.001127 + x_2 \cdot (0.1933)$

QN 7, contd.

final temperature $T_3 = T_{sat}$ (at $p = 100$ kpa.)

$= 99.632°c$.

$\therefore W_{total} = W_{1-2} + W_{2-3}$

$= P_1(v_2 - v_1) + 0$.

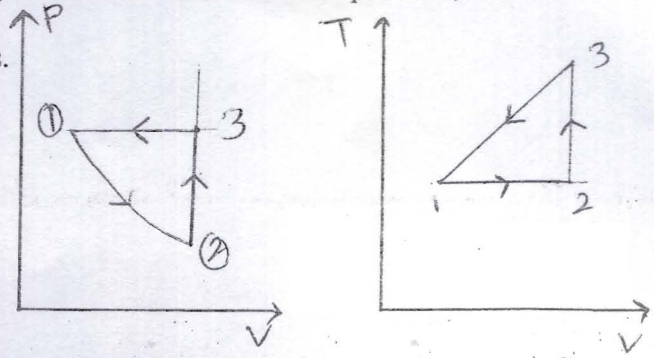
$= 1000(V_2 - V_1)$

$= 1000(0.02463 - 0.0985)$

$= -73.87$ KJ.

9. A gas undergoes a thermodynamic cycle consisting of the following three processes:
Process 1 − 2: expansion with $PV = constant$, $P_1 = 800\ kPa$, $U_2 = U_1$ **Process 2 − 3**: constant volume with $V_2 = V_3 = 2\ m3$, $U_3 - U_2 = 300\ kJ$ **Process 3 − 1**: constant pressure, $W_{31} = -1200\ kJ$

(a) Sketch the process on $P - V$ and $T - V$ diagrams.
(b) Calculate the net work for the cycle.
(c) Calculate the net heat for the cycle.
(d) Calculate the heat transfer for process 1 − 2.
(e) Calculate the heat transfer for process 3 − 1.
(f) Is this power cycle or a refrigeration cycle?



$P_1 V_1 = P_2 V_2$.

$U_2 - U_1 = 0$.

$V_2 = V_3 = 2 m^3$

$U_3 - U_2 = 300 \Rightarrow U_3 > U_2 \Rightarrow T_3 > T_2$.

$P_3 = P_1$

$W_{3-1} = -1200 kJ$.

for a cyclic process;

$\Sigma(du)_{1-3} = 0$.

$U_2 - U_1 + U_3 - U_2 + U_1 - U_3 = 0$.

$0 + 300 + U_1 - U_3 = 0$

$\therefore U_3 - U_1 = 300 kJ$.

$W_{3-1} = -1200$

$P_3(V_1 - V_3) = -1200$

$P_1(V_1 - 2) = -1200$

$800(V_1 - 2) = -1200$

$V_1 - 2 = -1.5$

$\therefore V_1 = 0.5 m^3$.

Again;

$P_1 V_1 = P_2 V_2$

$800 \times 0.5 = P_2 \times 2$

$\therefore P_2 = 200 kpa$.

QN9
Contd.

$W_{1-2} = P_1 V_1 \ln \frac{V_2}{V_1}$ $\quad W_{2-3} = 0. \quad W_{3-1} = -1200\,KJ.$

$= 800 \times 0.5 \ln \frac{2}{0.5}$

$= 554.517\ KJ.$

$\therefore W_{net} = W_{1-2} + W_{2-3} + W_{3-1}$

$= 554.517 + 0 + (-1200)$

$= -645.48\ KJ.$

$Q_{1-2} = W_{1-2} + U_2 - U_1$ $\quad Q_{2-3} = U_3 - U_2 + W_{2-3}$ $\quad Q_{3-1} = U_1 - U_3 + W_{3-1}$

$= 554.517 + 0$ $\quad = 300 + 0$ $\quad = -300 - 1200$

$Q_{1-2} = 554.517\ KJ.$ $\quad = 300\,KJ.$ $\quad = -1500\,KJ.$

$\therefore Q_{net} = Q_{1-2} + Q_{2-3} + Q_{3-1}$

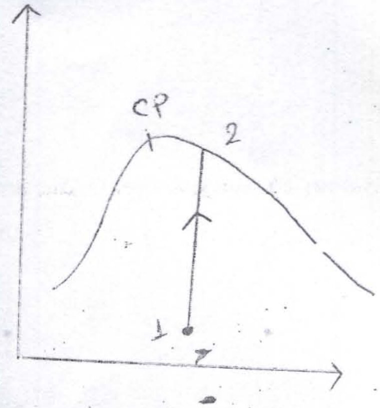
$= 554.517 + 300 - 1500$

$= 554.517 - 1200$

$Q_{net} = -645.48\ KJ.$

Since network is negative, it is refrigeration cycle.

10. A rigid vessel having a volume of 0.4 $m3$ initially contains a two-phase mixture at a pressure of 100 kPa with 2 % of its volume occupied by saturated liquid and the remaining by the saturated vapor. Heat is supplied to the vessel until it holds only saturated vapor. Determine the total heat transfer for the process.



Rigid vessel.

$V_1 = 0.4 m^3$

$P_1 = 100 kPa$

$V_L = 2\% \text{ of } V_1$

$V_g = 98\% \text{ of } V_2$

now;

$$x = \frac{m_g}{m_t}$$

$$= \frac{\frac{V_g}{v_g}}{\frac{V_g}{v_g} + \frac{V_L}{v_l}}$$

$$= \frac{\frac{98}{100}\frac{V_1}{1.6943}}{\frac{98}{100}\cdot\frac{V_1}{1.6943} + \frac{2}{100}\cdot\frac{V_1}{0.001043}}$$

$$= \frac{\frac{98}{1.6943}}{\frac{98}{1.6943} + \frac{2}{0.001043}}$$

$$x_1 = 0.0292808$$

$$v_1 = v_l + x_1 v_{lg}$$

$$= 0.001043 + 0.0292808\cdot(1.6933)$$

$$= 0.050624 \ m^3/kg.$$

$$u_1 = u_l + x_1 u_{lg}$$

$$= 417.41 + 0.050624\cdot(2088.3)$$

$$= 523.128 \ kJ/kg$$

6 N 10

Contd.

$$v_1 = \frac{V_1}{m}$$

$$m = \frac{V_1}{v_1}$$

$$= \frac{0.4}{0.050624}$$

$$= 7.9013 \text{ Kg.}$$

$$u_2 = (u_g)$$

Now;

$$v_2 = v_1 = v_g$$

v	u
0.05318	2602.3
0.04977	2601.3
0.050624	?

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$u_2 = 2602.3 + \frac{2601.3 - 2602.3}{0.04977 - 0.05318}(0.050624 - 0.05318)$$

$$= 2602.3 + \frac{(-1).(-2.556 \times 10^{-3})}{-3.41 \times 10^{-3}}$$

$$= 2601.55 \text{ KJ/Kg.}$$

$$\text{Heat transfer} = (u_2 - u_1).m$$

$$= (2601.55 - 523.128).(7.9013)$$

$$= 16422.23 \text{ KJ}$$

$$= 16.42223 \text{ MJ}$$ Ans.

15. Air ($0.1\ kg$) is contained in piston/cylinder assembly as shown in **Figure A1.15**. Initially, the piston rests on the stops and is in contact with the spring, which is in its unstretched position. The spring constant is $100\ kN/m$. The piston weighs $30\ kN$ and atmospheric pressure is $101\ kPa$. The air is initially at $300\ K$ and $200\ kPa$. Heat transfer occurs until the air temperature reaches the surrounding temperature of $700\ K$.

(a) Find the final pressure and volume.

(b) Find the process work.

(c) Find the heat transfer.

(d) Draw the P-V diagram of the process. [Take $R = 287\ J/kgK$ and $cv = 718\ J/kgK$]

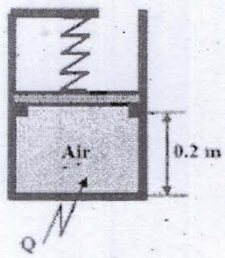
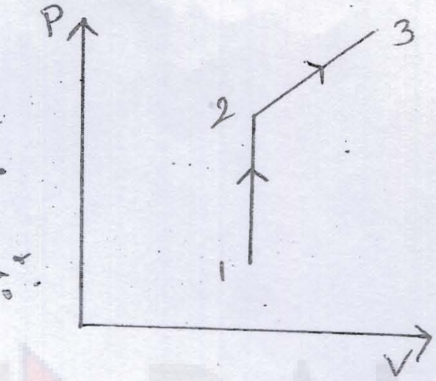
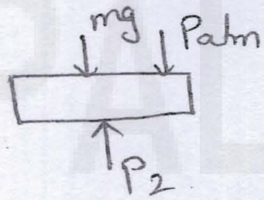


Figure A1.15





$m_g = 0.1\ kg$

$K = 100\ kN/m$

$W_p = 30\ kN$

$T_1 = 300\ K$

$P_1 = 200\ kPa$

$T_{final} = 700\ K$

$P_1 V_1 = m R T_1$

$200 \times 10^3 \times V_1 = 0.1 \times 287 \times 300$

$V_1 = 0.04305\ m^3$

Again; $V_1 = A \times h$

$0.04305 = A \times 0.2$

$\therefore A = 0.21525\ m^2$

Again;

$P_2 = \dfrac{mg}{A} + P_{atm}$

$= \dfrac{W_p}{A \times 10^3} + P_{atm}$

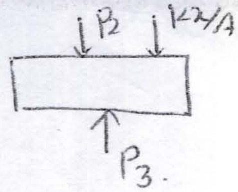
$= \dfrac{30}{0.21525} + 101$

$= 240.37\ kPa$

QN 15

Contd.

$$P_3 = P_2 + \frac{kx}{A}$$



$$= 240 + \frac{100 \times x}{0.21525}$$

$$P_3 = 240 + 464.57x$$

$$V_3 = (x + 0.2) \times 0.21525$$

$$\therefore P_3V_3 = mRT_3$$

$$(240 + 464.57x)(x + 0.2)\,0.21525 = \frac{0.1 \times 287 \times 700}{1000}$$

$$240x + 48 + 92.914x + 464.57x^2 = 93.333$$

$$\therefore 464.57x^2 + 332.914x - 45.333 = 0.$$

$$x = 0.11705 \text{ m}.$$

$$\therefore P_3 = 240 + 464.57 \times 0.11705$$

$$= 294.37 \text{ kpa}.$$

$$V_3 = 0.068245 \text{ m}^3.$$

$$W_{1\text{-}3} = W_{1-2} + W_{2-3}$$

$$= \frac{1}{2} \times (P_2 + P_3) \cdot (V_3 - V_1)$$

$$= \frac{1}{2} \times (240 + 294.37)(0.068245 - 0.04305)$$

$$= 6.7317 \text{ KJ}.$$

$$U_3 - U_1 = mc_v(T_3 - T_1)$$

$$= 0.1 \times 718 (700 - 300)$$

$$= 28.72 \text{ KJ}.$$

$$Q_{1-3} = W_{1-3} + (U_3 - U_1)$$

$$= 35.4517 \text{ KJ}.$$